

# A PARTICLE PICTURE OF GRAVITATIONAL FIELDS

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**Abstract.** In this paper, by using mathematical interpretations on space-time obtained in [Mo3], we shall give a candidate for a particle picture of continuous gravitational fields.

## 1. SPECIAL RELATIVITY

First, let us review mathematical interpretations on space-time obtained in [Mo3]. For the inertial frame of reference  $S$  on the ground, consider the inertial frame of reference  $S'$  moving at the constant speed  $V$  e.g. a railway bogie. Here, assume that the stand AB of this bogie is parallel to the direction of motion of  $S'$  and that a light equipment is put on the midpoint of AB. Let us turn on the light. Then, it is no wonder that a person on the bogie  $S'$  will see that the light arrives at the points A and B at the same time. On the other hand, a person on the ground  $S$  will see that, by the principle of invariant light speed, the light arrives at the point A faster than at the point B.

Stationary  $S$ : A  $\xleftarrow{\text{light}}$   $\xrightarrow{\text{light}}$  B    Moving  $S'$ : A  $\xleftarrow{\text{light}}$   $\xrightarrow{\text{light}}$  B

Although turning on the light is the only one event, this example shows that the utterly different worlds exist associated to the different inertial frames of reference. Based on this, we denoted in [Mo3] that our world consists of each contribution from a world associated to a inertial frame of reference with different momentum and that all substances have the property of a wave and a particle at once.

## 2. GENERAL RELATIVITY

**2.1. Review of the classical theory.** General relativity due to Einstein says that the gravitational field is caused by the distortion of space-time and that its equation is given by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}.$$

Here, in the left hand side, the first term is the Einstein tensor which represents the distortion of space-time and the second term is the cosmological constant term. On the other hand, in the right hand side,  $\kappa$  is the Einstein's constant of

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gravitation and  $T_{\mu\nu}$  is the energy-momentum tensor which represents the distribution of the matter field.

**2.2. A particle picture of gravitational fields.** A particle picture of gravitational fields is nothing but the discretization of space-time and it is also an interesting matter from the standpoint of number theory. Now, we shall give a candidate for the particle picture of gravitational fields based on the principle that our world consists of each contribution from a world associated to a inertial frame of reference with different momentum.

2.2.1. *Preliminary.* Let  $S_0$  denote a stationary inertial frame of reference of a matter field and write the Einstein's equations at  $S_0$  as  $G_{\mu\nu}^{(0)} + \Lambda g_{\mu\nu}^{(0)} = \kappa T_{\mu\nu}^{(0)}$ . On the other hand, let  $S_V$  denote a inertial frame of reference moving with momentum  $V$  and write the Einstein's equations at  $S_V$  as

$$(*) \quad G_{\mu\nu}^{(V)} + \Lambda g_{\mu\nu}^{(V)} = \kappa T_{\mu\nu}^{(V)}.$$

2.2.2. *Definition.* Consider the Einstein-Hilbert action of each equation  $(*)$  and put these actions together. We expect that this new action should give a good equation for the particle picture of gravitational fields. The left hand side of this new equation would represent the distortion of space-time and the right hand side the distribution of the matter field.

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