NOTES ON THE PARTICLE SPIN

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Abstract. The nature of the spin, which is believed to link matter, force and space, is still largely unexplained, even for fundamental things. In this essay, I would like to give a consideration of the spin from a simple point of view.

1. Statistics

Spin quantum numbers are broadly classified into boson particles according to Bose-Einstein statistics and fermion particles according to Fermi statistics. That is, a boson particle is one in which several particles can occupy the same quantum state, and a fermion particle is one in which no more than two particles can occupy the same quantum state. The fermion particles in the single-particle state represent so-called matter, and according to the gauge principle, the boson particles mediate the forces between such matters.

In the primitive universe, when matter and antimatter existed in the same degree, they quickly annihilated each other and it is difficult to determine which one represents matter and which one represents force. To use an analogy in the form of a Feynman diagram, the people playing catch are the fermion particles and the ball is the boson particle, and it is not possible to identify either the catcher or the ball as mediating the force. In this section, we disregard which is the matter and which is the force, and consider which combination is optimal, assuming that there are only two statistics, Bose-Einstein statistics and Fermi statistics.

In the analogy above, let A denote the one playing catch and B denote the ball. We can consider the four possible combinations

$$(\text{A, B})=(\text{fermion, fermion}),\ (\text{boson, fermion}),$$
$$(\text{boson, boson}),\ (\text{fermion, boson}).$$

(i) (A, B)=(fermion, fermion)

Since B is a fermion particle and there is only one particle in the system that fits between A, there is almost no fundamental interaction and the system is not suitable for real situations.

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(ii) (A, B) = (boson, fermion)

This is also not suitable for real situations for the same reason as (i).

(iii) (A, B)=(boson, boson)

There will be a large number of boson particles in the system satisfying the combination of A and B, which is the exact opposite of the situation in (i) and (ii), where the fundamental interaction occurs in a chain, and is not suitable for the real situation.

(iv) (A, B) = (fermion, boson)

B is a boson particle, and there are a large number of particles in the system that fit between A, but few fermion particles that fit into A. Thus, the fundamental interactions do not occur rarely, nor do they occur in a chain and it is the most realistic combination of the remaining possibilities. When focusing on two fermion particles in close proximity, this mechanism can also explain Pauli exclusion principle, which states that the fermion particles do not favour identical quantum states.

When (A, B)=(fermion, boson), if the boson particle does not annihilate after the fundamental interaction, the fundamental interaction is effectively prevented by the reversible fundamental interaction. On the other hand, if the fermion particles also annihilate immediately, the world as we know it today would not exist. In the following, we will consider from the difference in the spin quantum numbers that the fermion particles are relatively stable and the boson particles disappear as soon as the fundamental interaction occurs as virtual particles.

2. Spins of particles

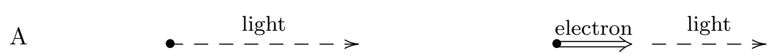
It follows from the quantum field theory that the spin quantum numbers of boson particles are integers and those of fermion particles are half-integers. Although the spin quantum numbers of electrons are calculated as angular momenta, they have been considered to be due to the internal structure of electrons, rather than to their actual rotation. Here, we would like to deepen the discussion using relativity theory described in [M], consider the spin as being due to rotation and further discuss the difference between the spin quantum numbers of boson particles and fermion particles, as described in the section above.

2.1. **Spin and rotation.** It was argued in [M] that each inertial frame moving from a free electron at various speeds (momentum) has its own world, and the momentum expansion of the field operator obtained from the Schrödinger equation can be interpreted mathematically as the annihilation or creation of a single particle by adding up the contributions from these different worlds. The fact that each inertial frame moving at different speeds (momentum) has its own world is a consequence of the principle of variable light speed.

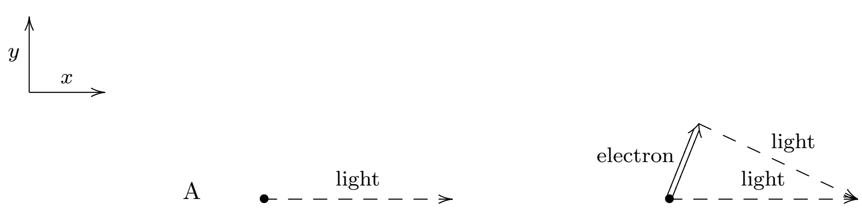
Furthermore, taking into account that the photon has spin quantum number one, these worlds do not simply exist linearly, but exist both linearly and rotationally. The contributions from these worlds add up to form a wave motion of the electron, which gives the electron spin. In other words, the electron itself is not spinning, but the space is spinning with the spin of the photon.

2.2. **Differences in spin quantum numbers between boson and fermion.** The spin quantum number of the boson particle is an integer, whereas the spin quantum number of the fermion particle is a half-integer, which is puzzling. Moreover, as mentioned in the first section, fermion particles are relatively stable in the present world, while boson particles, as virtual particles, disappear as soon as the fundamental interaction occurs.

2.2.1. *Principle of variable light speed.* We begin by restating the discussion of the principle of variable light speed given in [M] as applied to electrons with relativistic effects.



As shown in the above figure, light emitted from point A, which is stationary on the ground, and light emitted from an electron moving with momentum $\mathbf{p}$ with respect to the ground, both appear to be travelling at the speed of light c and the formula for the composition of velocities (vectors) does not hold. It may be one idea to assume that this is a property of light, but we would like to mathematically set up a hypothetical space in which the formula for the composition of the velocities (vector) is valid.



Here, a person on the ground can see only the x-axis and the formula of vectors is concluded in this mathematically virtual world (xy-plane) which is invisible to the person. Whatever momentum $\mathbf{p}$ the electron travels with, from an observer stationary on the ground, the light emitted from the electron always appears to be travelling at light speed c. As a matter of course, the faster the speed of the electron is, the more it vanishes in the direction of the y-coordinate, and the less it contributes to observation from point A.

2.2.2. *Momentum expansion of the Dirac field.* Displaying the momentum expansion of the conjugate $\bar{\psi}(x)$ of the Dirac field satisfied by an electron with mass

m, we obtain

$$\bar{\psi}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3 \cdot 2\sqrt{\mathbf{p}^2 + m^2}} \sum_{s=\pm\frac{1}{2}} [c^\dagger(\mathbf{p}, s)\bar{u}(\mathbf{p}, s)e^{i\mathbf{p}\cdot x} + d(\mathbf{p}, s)\bar{v}(\mathbf{p}, s)e^{-i\mathbf{p}\cdot x}].$$

where $c^{(\dagger)}(\mathbf{p}, s)$ and $d^{(\dagger)}(\mathbf{p}, s)$ denote the creation and annihilation operators for particles and antiparticles respectively, s is the spin, $\bar{u}(\mathbf{p}, s)$ and $\bar{v}(\mathbf{p}, s)$ are the spinors which are the coefficients. Each inertial frame moving at different speeds (momenta) has its own world, and as can be seen from this momentum expansion, the contribution from the momentum $\mathbf{p}$ is

$$\frac{1}{(2\pi)^3 \cdot 2\sqrt{\mathbf{p}^2 + m^2}} \sum_{s=\pm\frac{1}{2}} [c^\dagger(\mathbf{p}, s)\bar{u}(\mathbf{p}, s)e^{i\mathbf{p}\cdot x} + d(\mathbf{p}, s)\bar{v}(\mathbf{p}, s)e^{-i\mathbf{p}\cdot x}].$$

Combining this indication with the discussion in subsection 2.2.1, it can be interpreted that for the production of an electron with momentum $\mathbf{p}$, a corresponding antiparticle with momentum $\mathbf{p}$ is produced in the virtual y-coordinate direction set in 2.2.1.

2.2.3. *Spin quantum number.* The above discussion shows that, depending on the degree of the existence of the particle and the existence of the antiparticle in the virtual world, it is possible for the spin quantum number of the fermion particle to be half-integer, even if the spin is due to rotation. On the other hand, in the same way for boson particles, there is a field momentum expansion, and the difference in the spin quantum numbers of boson and fermion particles depends on the degree of the existence of the particle and the existence of the antiparticle in the virtual world. The instability of the boson particle as a virtual particle is due to annihilation with this antiparticle, and the difference in stability with the fermion particle also depends on this degree.

References

[M] K. Morita.:*On the mathematical interpretation of the quantum field theory.* (available at https://kazuma-morita.jimdofree.com/)